

### POZNAN UNIVERSITY OF TECHNOLOGY

**EUROPEAN CREDIT TRANSFER AND ACCUMULATION SYSTEM (ECTS)** 

### **COURSE DESCRIPTION CARD - SYLLABUS**

Course name

Mathematics [S1IMat1>MAT1]

Course

Field of study Year/Semester

Materials Engineering 1/1

Area of study (specialization) Profile of study

general academic

Level of study Course offered in

first-cycle Polish

Form of study Requirements full-time compulsory

**Number of hours** 

Lecture Laboratory classes Other (e.g. online)

45 0

Tutorials Projects/seminars

30 0

Number of credit points

7,00

Coordinators Lecturers

dr Marek Adamczak dr Marek Adamczak

marek.adamczak@put.poznan.pl marek.adamczak@put.poznan.pl

## **Prerequisites**

A student has the basic knowledge of mathematics at the secondary level. The student performs the elementary algebraic operations and solves simple equations and inequalities. The student is aware of the need for further education.

# Course objective

The main aim of this course is to acquaint the student with the basic concepts of higher mathematics. After the course the student will be able to: perform the basic operations on matrices and vectors, find the limit of a given sequence of real numbers, plot graphs of the elementary functions, find the limit of a given function, find the derivative of a given function, use the methods of a curve analysis, find the indefinite and the definite integral of a given function, perform the algebraic operations on complex numbers, examine the convergence of a given series of real numbers, find the partial derivatives of a given function, determine the extreme points of a given functions of two variables.

# Course-related learning outcomes

#### Knowledge:

1. the student has the knowledge in mathematics including selected sections of mathematical analysis,

algebra and analytic geometry.

#### Skills:

1. the student is able to use mathematical methods in the analysis of technical problems.

## Social competences:

1. the student is aware of the need to deepen and expand knowledge.

### Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Learning outcomes presented above are verified as follows:

#### Tutorials:

The skills acquired during the classes are verified on the basis of two written tests. To pass the classes it is necessary to get at least 50% of the total number of points from the mentioned written tests. Lecture:

The knowledge acquired during the lecture is verified on the basis of a written examination. To pass the lecture it is necessary to get at least 50% of the points from the mentioned written examination. Grading system:

- 0%-50% 2.0,
- 50%-60% 3.0,
- 60%-70% 3.5,
- 70%-80% 4.0,
- 80%-90% 4.5.
- 90%-100% 5.0.

## Programme content

Matrix algebra. Vectors. Sequence numbers. Function of one variable. Differential and integral calculus of functions of one variable. Complex numbers. Number series. Functions of two variables. Differential calculus of functions of two variables. Power series.

## **Course topics**

#### Lecture:

- matrix operations and determinants a Cartesian product of sets, definition of a matrix, basic types of matrices, elementary operations on matrices (addition, subtraction, multiplication by a constant, multiplication, transposition), definition of a matrix determinant, a singular and a non-singular matrix, the rule for finding the determinant of a 2x2 matrix, the rule of Sarrus for finding the determinant of a 3x3 matrix, the Laplace expansion, properties of matrix determinants,
- vectors definition of a vector, elementary operations on vectors (addition, subtraction, multiplication by a constant), a length of a vector, a scalar product of vectors, a vector product (cross product) of vectors, a perpendicularity and a collinearity of vectors, the formula for the surface area of a triangle formed by vectors, the formula for the surface area of a parallelogram formed by vectors, a mixed product (scalar triple product) of vectors, the formula for the volume of a parallelepiped formed by vectors, the formula for the volume of a tetrahedron formed by vectors, a complementarity of vectors,
- sequences of real numbers definition of a sequence of real numbers, a boundedness of a sequence of real numbers, a monotonicity of a sequence of real numbers, definition of a limit of a sequence of real numbers, properties of the limits of sequences of real numbers, the Squeeze theorem, Euler's number,
- function of one variable definition of a function, a domain of a function, a counter-domain of a function, a set of values of a function, a graph of a function, a periodic function, an even function, an odd function, a boundedness of a function, a monotonicity of a function, a composition of functions, a surjection and an injection, an inverse function, elementary functions, limits of functions, properties of the limits of functions, a continuity of a function, properties of continuous functions, definition of a derivative of a function and its properties, L"Hôpital"s rule, basic steps of a curve analysis (asymptotes, extreme points, monotonicity intervals, a minimum value, a maximum value, a concavity, a convexity, inflection points), a differential of a function,
- indefinite integral an antiderivative, definition of an indefinite integral, basic rules of the integral calculus, properties of the indefinite integral, an integration by substitution, an integration by parts, an integral of a rational function, an integral of an irrational function, a conversion of trigonometric

integrals to rational integrals,

- definite integral the Riemann sum, the Riemann integral, properties of the definite integral, the Newton-Leibniz formula, an integration by substitution, an integration by parts, the mean value theorem for definite integral, an integral of an even function, an integral of an odd function, an integral of an periodic function, a surface area of a surface bounded by curves, a length of an arc, a surface area of the surface obtained by a rotation of the graph of a function around an axis, a volume of the solid obtained by a rotation of the graph of a function around an axis, improper integrals,
- complex numbers definition of a complex number, an algebraic form of a complex number, elementary operations on complex numbers, a complex conjugate of a complex number, a modulus of a complex number, an argument of a complex number, a main argument of a complex number, a trigonometric form of a complex number, de Moivre's formula, a root of a complex number, an exponential form of a complex number,
- series of real numbers definition of a series of real numbers, a convergence of a series of real numbers, properties of the convergent series of real numbers, a necessary condition for the convergence of a series of real numbers, convergence tests of series of real numbers (the direct comparison test, the limit comparison test, the ratio test, the root test, the integral test), definition of an alternating series, the alternating series test, an absolute and a conditional convergence of a series of real numbers.
- functions of two variables definition of a function of two variables, functions and graphs of basic surfaces (a plane, a paraboloid, a sphere, a cone, a cylinder), definition of a partial derivative, the symmetry of second derivatives, higher order partial derivatives, an exact differential, extremes of functions of two variables,
- power series definition of a power series, a radius of the convergence of a power series, an interval of the convergence of a power series, the Taylor series, the Maclaurin series. Tutorials:
- matrix operations and determinants performing basic operations on matrices (addition, subtraction, multiplication by a constant, multiplication, transposition), finding the determinant of a given matrix (the rule for finding the determinant of a 2x2 matrix, the Rule of Sarrus for finding the determinant of a 3x3 matrix, the Laplace expansion),
- vectors calculating the length of a given vector, calculating the scalar product of given vectors, calculating the angle between given vectors, examination the perpendicularity of given vectors, examination the collinearity of given vectors, calculating the vector product of given vectors, calculating the surface area of a triangle formed by given vectors, calculating the surface area of a parallelogram formed by given vectors, calculating the mixed product of given vectors, calculating the volume of a parallelepiped formed by given vectors, calculating the volume of a tetrahedron formed by given vectors, examination the complementarity of given vectors,
- sequences of real numbers examination the boundedness of a given sequence of real numbers, examination the monotonicity of a given sequence of real numbers, calculation the limit of a given sequence of real numbers using the basic properties of limits of sequences of real numbers, calculation the limit of a given sequence of real numbers using the Squeeze theorem, calculating the limit of a given sequence of real numbers associated with the Euler number,
- function of one variable finding the domain of a given function, finding the set of values of a given function, drawing the graph of a given function, calculation the limit of a given function, calculating the one-sided limits of a given function, examination the continuity of a given function, finding the derivative of a given function, calculating the limit of a given function using the L"Hospital rule, the curve analysis (finding of asymptotes, extreme points, monotonicity intervals, concavity intervals, convexity intervals, inflection points, and drawing the graph),
- indefinite integral finding the indefinite integral of a given function (application of basic rules of the integral calculus, the integration by substitution method, the integration by parts method), finding the integral of a given rational function, finding the integral of a given trigonometric function,
- definite integral application of the Newton-Leibniz formula, calculating the definite integral of a given function by the integration by substitution method, calculating the definite integral of a given function by the integration by parts method, calculating the surface area of the surface bounded by given curves, calculating the length of a given arc, calculating the surface area of the surface obtained by a rotation of the graph of a given function around the axis, calculating the volume of the solid obtained by a rotation of the graph of a given function around the axis,
- complex numbers performing operations on complex numbers in the algebraic form, calculating the modulus of a given complex number, finding the argument of a given complex number, finding the main argument of a given complex number, conversion of a given complex number in the algebraic form to

the trigonometric form, multiplication and division of complex numbers in the trigonometric form, raising of a given complex number to a given power (application of de Moivre's formula), finding the root of a given complex number, finding the solutions to a given equation in the complex numbers solutions set,

- series of real numbers examination the convergence of a given series of real numbers (application of the necessary condition, the direct comparison test, the ratio test, and the root test), examination the convergence of a given alternating series by the alternating series test,
- functions of two variables finding the partial derivatives of a given function of two variables, finding the exact differential of a given function of two variables, finding the local extrema of a given function of two variables.

# **Teaching methods**

Lecture: traditional lectures (theory presented in connection with the current knowledge of students). Tutorials: blackboard tutorials (solving of math problems with the help of a teacher).

# **Bibliography**

#### Basic

- 1. W. Krysicki, L. Włodarski, Analiza matematyczna w zadaniach 1, Wydawnictwo Naukowe PWN, Warszawa, 2018.
- 2. W. Krysicki, L. Włodarski, Analiza matematyczna w zadaniach 2, Wydawnictwo Naukowe PWN, Warszawa, 2018.
- 3. M. Gewert, Z. Skoczylas, Analiza matematyczna 1: definicje, twierdzenia, wzory, Oficyna Wydawnicza GiS, Wrocław, 2019.
- 4. M. Gewert, Z. Skoczylas, Analiza matematyczna 2: definicje, twierdzenia, wzory, Oficyna Wydawnicza GiS, Wrocław, 2019.
- 5. T. Jurlewicz, Z. Skoczylas, Algebra i geometria analityczna: definicje, twierdzenia, wzory, Oficyna Wydawnicza GiS, Wrocław, 2012.

#### Additional

- 1. M. Gewert, Z. Skoczylas, Analiza matematyczna 1: przykłady i zadania, Oficyna Wydawnicza GiS, Wrocław, 2017.
- 2. M. Gewert, Z. Skoczylas, Analiza matematyczna 2: przykłady i zadania, Oficyna Wydawnicza GiS, Wrocław 2019.
- 3. T. Jurlewicz, Z. Skoczylas, Algebra i geometria analityczna: przykłady i zadania, Oficyna Wydawnicza GiS, Wrocław, 2015.

# Breakdown of average student's workload

	Hours	ECTS
Total workload	175	7,00
Classes requiring direct contact with the teacher	75	3,00
Student's own work (literature studies, preparation for laboratory classes/tutorials, preparation for tests/exam, project preparation)	100	4,00